experimentally, in order to completely describe the pressure-time history in the compression chamber, and hence, the pressure-release time T_R.

In an effort to express the time constant θ_1 analytically, the following analysis is made. A general expression for the equation of motion of a viscous, compressible fluid is derived in reference (c) and can be expressed in vector form as

$$\frac{\rho d\overline{v}}{dt} = \rho\overline{F} - \nabla P - \tfrac{2}{3}\nabla\left[\mu(\nabla\cdot\ \overline{v})\right] + \left[\nabla\cdot\ (2\mu\nabla)\right]\overline{v} + \nabla x(\mu\overline{\zeta}) \qquad (7)$$

where the nomenclature is

ρ . . . mass density of the fluid, slugs/in³

$\overline{v}$ . . . velocity vector, in/sec

P . . . fluid pressure, psia

$\overline{F}$ . . . body forces per unit mass, lb/slug

μ . . . coefficient of viscosity of fluid, lb-sec/in²

$\overline{\zeta}$ . . . vorticity vector, sec^{-1}

∇ . . . operator del

The vorticity vector $\overline{\zeta}$ is defined as the curl of the velocity vector

$$\overline{\zeta} = \nabla x\overline{v} \qquad (8)$$

Combining (8) with (7) and performing the operations indicated by the operator ∇, we have

$$\frac{\rho d\overline{v}}{dt} = \rho\overline{F} - \nabla P - \mu\nabla x\overline{\zeta} + \tfrac{4}{3}\mu\nabla(\nabla\cdot\ \overline{v})$$

$$+ 2(\nabla\cdot\nabla\mu)\ \overline{v} + \nabla\mu x\overline{\zeta} - \tfrac{2}{3}\ \mu(\nabla\cdot\ \overline{v}) \qquad (9)$$

If we invoke the condition of incompressibility

$$\nabla \cdot \overline{v} = 0 \tag{10}$$

and if we consider the coefficient of viscosity to be independent of the spatial coordinates

$$\nabla \mu = 0 \tag{11}$$

then equation (9) is simplified to the form

$$\rho \frac{d\overline{v}}{dt} = \rho \overline{F} - \nabla P - \mu \nabla x \overline{\zeta} \tag{12}$$

Combining this with equation (8), we have

$$\frac{d\overline{v}}{dt} = \overline{F} - \frac{1}{\rho}\nabla P + \nu \nabla^2 \overline{v} \tag{13}$$

where

ν . . . is kinematic viscosity of fluid = $\frac{\mu}{\rho}$, in^2/sec

Expanding equation (13) into its components in cylindrical coordinates, we obtain the familiar Navier-Stokes equations for a viscous, incompressible fluid

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r^2} = F_1 - \frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$+ \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \tag{14a}$$